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Classical Description of the Radiation of a Charged Particle in a Strong-Laser Plasma

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The behavior of a charged particle in a strong-laser plasma is discussed by solving the generally covariant equation of motion for a charged particle. The classical description for the radiation of a charged particle in a strong-laser plasma is given, and the intensity and the radiation power are derived in detail.

1. INTRODUCTION

In Zhu *et al.* (1995) a laser plasma is described by Riemannian geometry. The correctness of this description has not been proved. Because an electron or other charged particle is a microparticle, it can be taken as a probe to test the geometry. We know that a charged particle in a vacuum can neither emit nor absorb radiation, because the laws of energy and momentum conservation cannot be satisfied at the same time. Only when a charged particle accelerates under the action of an external field can radiation be emitted. Such a field could be a Coulomb field in an atom (in which case the emission is the bremsstrahlung) or a bending magnetic field in a high-energy accelerator (the synchrotron radiation) or a plane-wave electromagnetic field (Sarachik and Schappert, 1970). Therefore, one of the methods to test this geometry is to measure the radiation of a charged particle in a strong-laser plasma. In this paper, we give a classical description for the radiation of a charged particle. Quantum considerations will be given elsewhere.

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2. MOTION OF A CHARGED PARTICLE IN A STRONG-LASER PLASMA

2.1. Equation of Motion

In Zhu *et al.* (1995) we gave a Riemannian geometrical description of a strong-laser plasma. For a one-dimensional isothermally expanding plasma along the x direction, the motion of a charged particle in a strong-laser plasma obeys the generally covariant equation (Weinberg, 1972)

$$\frac{DU^\mu}{D\tau} = \frac{e}{mc^2}\,\bar{g}^{\mu\nu}F_{\nu\rho}U^\rho \tag{1}$$

where $U^\mu = dx^\mu/d\tau$ is the four-dimensional velocity of the electron, τ is the proper time, $DU^\mu/D\tau$ is the covariant derivative of U^μ with respect to τ, $\bar{g}^{\mu\nu}$ is the optical metric of the strong-laser plasma, $F_{\mu\nu}$ is the electromagnetic field tensor, e and m are the charge and mass of a charged particle, and c is the velocity of light. From Zhu *et al.* (1995) we have

$$\bar{g}_{00} = -1/(1 - N) \tag{2}$$

$$\bar{g}_{01} = c_s N_s/c(1 - N) \tag{3}$$

$$\bar{g}_{11} = \bar{g}_{22} = \bar{g}_{33} = 1 \tag{4}$$

$$N = n/n_c \tag{5}$$

$$N_s = n_s/n_c \tag{6}$$

$$n_c = m\omega^2/4\pi e^2 \tag{7}$$

$$E_y = F_{20} = -\frac{\partial A_2}{\partial x^0} = \frac{m\omega v_e}{e}A(\xi)\cos\omega t \tag{8}$$

$$B_z = F_{12} = \frac{\partial A_2}{\partial x^1} = -\frac{ckmv_e}{e}\frac{dA(\xi)}{d\xi}\sin\omega t \tag{9}$$

A_2 is the third component of the four-dimensional electromagnetic vector

$$A_2 = -\frac{cmv_e}{e}A(\xi)\sin\omega t \tag{10}$$

The other components of the four-dimensional electromagnetic vector are zero, i.e.,

$$A_0 = A_1 = A_3 = 0 \tag{11}$$

Thus equation (1) can be written in the form

$$\frac{DU^0}{D\tau} = \frac{e}{mc^2}(\bar{g}^{00}F_{02}U^2 + \bar{g}^{01}F_{12}U^2) \tag{12}$$

$$\frac{DU^1}{D\tau} = \frac{e}{mc^2}(\bar{g}^{10}F_{02}U^2 + \bar{g}^{11}F_{12}U^2) \tag{13}$$

$$\frac{DU^2}{D\tau} = \frac{e}{mc^2}(\bar{g}^{22}F_{20}U^0 + \bar{g}^{22}F_{21}U^1) \tag{14}$$

$$\frac{DU^3}{D\tau} = 0 \tag{15}$$

2.2. Solution of the Equation

Since (Zhu *et al.*, 1995)

$$\Gamma^3_{\mu\nu} = 0, \tag{16}$$

equation (15) is reduced to

$$\frac{dU^3}{d\tau} = 0 \tag{17}$$

For the initial condition $U^3|_{A=0} = 0$, the solution of equation (16) is

$$U^3 = 0 \tag{18}$$

Since $\Gamma^2_{\mu\nu} = 0$ (Zhu *et al.*, 1995), equation (14) is reduced to

$$\frac{dU^2}{d\tau} = \frac{e}{mc^2}\bar{g}^{22}\left(-\frac{\partial A_2}{\partial x^0}\frac{dx^0}{d\tau} - \frac{\partial A_2}{\partial x^1}\frac{dx^1}{d\tau}\right)$$

$$= -\frac{e}{mc^2}\bar{g}^{22}\frac{dA_2}{d\tau} \tag{19}$$

For the initial condition $U^2|_{A=0} = 0$ and $\bar{g}^{22} = 1$ the solution of equation (19) is

$$U^2 = -\frac{eA_2}{mc^2} = \frac{v_e A(\xi)\sin\omega t}{e} \tag{20}$$

From the line element $d\tau^2 = \bar{g}_{\mu\nu}\, dx^\mu\, dx^\nu$, we obtain

$$\bar{g}_{\mu\nu}U^\mu U^\nu = -1 \tag{21}$$

The nonzero components of $\bar{g}_{\mu\nu}$ are $\bar{g}_{00}$, $\bar{g}_{01}$, $\bar{g}_{11}$, $\bar{g}_{22}$, and $\bar{g}_{33}$, and the equation (21) can be written as

$$\bar{g}_{00}(U^0)^2 + 2\bar{g}_{01}U^0U^1 + \bar{g}_{11}(U^1)^2 + \bar{g}_{22}(U^2)^2 = -1 \tag{22}$$

By taking the covariant differentiation of equation (22) with respect to τ, we obtain

$$\bar{g}_{00}U^0 \frac{DU^0}{D\tau} + \bar{g}_{01}U^1 \frac{DU^0}{D\tau} + \bar{g}_{01}U^0 \frac{DU^1}{D\tau} + \bar{g}_{11}U^1 \frac{DU^1}{D\tau} + \bar{g}_{22}U^2 \frac{DU^2}{D\tau} = 0 \tag{23}$$

Substituting equations (12) and (13) into equation (23) yields

$$[\bar{g}_{00}\bar{g}^{00}F_{02} + \bar{g}_{00}\bar{g}^{01}F_{12} + \bar{g}_{01}\bar{g}^{01}F_{02} + \bar{g}_{01}\bar{g}^{11}F_{12}]U^0 + [\bar{g}_{01}\bar{g}^{00}F_{02} + \bar{g}_{01}\bar{g}^{01}F_{12} + \bar{g}_{11}\bar{g}^{01}F_{02} + \bar{g}_{11}\bar{g}^{11}F_{12}]U^1 + \frac{mc^2}{e}\bar{g}_{22}\frac{dU^2}{d\tau} = 0 \tag{24}$$

It is easy to see that

$$U^1 = \frac{dx^1}{d\tau} = \frac{dx^1}{dx^0}\frac{dx^0}{d\tau} = \frac{v_x}{c}U^0 \tag{25}$$

$$\frac{dU^2}{d\tau} = \frac{dU^2}{dx^0}\frac{dx^0}{d\tau} = \frac{dU^2}{dx^0}U^0 \tag{26}$$

and from equation (20), we have

$$\frac{dU^2}{dx^0} = \frac{\omega v_e}{c^2}A(\xi)\cos\omega t \tag{27}$$

By substitution of equations (25)–(27) into equation (24), we obtain the component of the three-dimensional velocity

$$v_x = -c\left[(\bar{g}_{00}\bar{g}^{00} + \bar{g}_{01}\bar{g}^{01} - 1)A(\xi)\cos\omega t + (\bar{g}_{00}\bar{g}^{01} + \bar{g}_{01}\bar{g}^{11})\frac{ck}{\omega}\frac{dA(\xi)}{d\xi}\sin\omega t\right] \times \left[(\bar{g}_{01}\bar{g}^{00} + \bar{g}_{11}\bar{g}^{01})A(\xi)\cos\omega t + (\bar{g}_{01}\bar{g}^{01} + \bar{g}_{11}\bar{g}^{11})\frac{ck}{\omega}\frac{dA(\xi)}{d\xi}\sin\omega t\right]^{-1} \tag{28}$$

Inserting equations (20) and (25) into equation (22), we get the first component of the four-dimensional velocity

$$U^0 = \left(-\left\{1 + \left[\frac{v_e}{c} A(\xi) \sin \omega t\right]^2\right\} \times \left[\bar{g}_{00} + 2\bar{g}_{01} \frac{v_x}{c} + \bar{g}_{11}\left(\frac{v_x}{c}\right)^2\right]^{-1}\right)^{1/2} \tag{29}$$

From equation (21), we obtain the second component of the four-dimensional velocity

$$U^1 = \left(-\left\{1 + \left[\frac{v_e}{c} A(\xi) \sin \omega t\right]^2\right\} \times \left[\bar{g}_{00} + 2\bar{g}_{01} \frac{v_x}{c} + \bar{g}_{11}\left(\frac{v_x}{c}\right)^2\right]^{-1}\right)^{1/2} \frac{v_x}{c} \tag{30}$$

where v_x is given by equation (28). The other two components of the four-dimensional velocity are given by equations (20) and (18), and the y and z components of the three-dimensional velocity are

$$\begin{aligned} v_y &= cU^2/U^0 \\ &= v_e A(\xi) \sin \omega t \left(-\left[\bar{g}_{00} + 2\bar{g}_{01} \frac{v_x}{c} + \bar{g}_{11}\left(\frac{v_x}{c}\right)^2\right] \times \left\{1 + \left[\frac{v_e}{c} A(\xi) \sin \omega t\right]^2\right\}^{-1}\right)^{1/2} \end{aligned} \tag{31}$$

$$v_z = cU^3/U^0 = 0 \tag{32}$$

From equations (28) and (31) we see that the velocity components v_x and v_y are the functions of time, i.e., a charged particle is accelerated in a strong-laser plasma. So a charged particle in a strong-laser plasma can emit radiation.

3. RADIATION

3.1. Radiation Field

The radiation intensity emitted by an accelerated charged particle has the form (Jackson, 1976)

$$\mathbf{E} = \frac{e}{c}\left\{\frac{1}{k^3 r}\mathbf{r}_1 \times [(\mathbf{r}_1 - \beta) \times \dot{\beta}]\right\} \tag{33}$$

$$\mathbf{B} = \frac{e}{c}\left\{\frac{1}{k^3 r}[\mathbf{r}_1 \times (\beta \times \dot{\beta}) + \dot{\beta}] \times \mathbf{r}_1\right\} \tag{34}$$

with

$$r = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2} \tag{35}$$

$$k = 1 - \mathbf{r}_1 \cdot \beta \tag{36}$$

$$\beta = \mathbf{v}/c \tag{37}$$

where (x_0, y_0, z_0) are the space coordinates of the charged particle, $\mathbf{r}_1$ is the unit vector in the direction of $\mathbf{r}$, $\mathbf{v}$ is the velocity of the charged particle, and $\dot{\beta}$ is the derivative of β with respect to time.

We now consider the radiation emitted by a free electron in a strong-laser plasma and rewrite its velocity components as

$$v_x = -c(a \cos \omega t + b \sin \omega t)/(f \cos \omega t + g \sin \omega t) \tag{38}$$

$$v_y = cd \sin \omega t\left\{-\left[\bar{g}_{00} + 2\bar{g}_{01}\frac{v_x}{c} + \bar{g}_{11}\left(\frac{v_x}{c}\right)^2\right] \times [1 + (h \sin \omega t)^2]^{-1}\right\}^{1/2} \tag{39}$$

where

$$a = (\bar{g}_{00}\bar{g}^{00} + \bar{g}_{01}\bar{g}^{01} - 1)A(\xi_0) \tag{40}$$

$$b = (\bar{g}_{00}\bar{g}^{01} + \bar{g}_{01}\bar{g}^{11})\frac{ck}{\omega}\frac{dA(\xi)}{d\xi}\bigg|_{\xi=\xi_0} \tag{41}$$

$$f = (\bar{g}_{01}\bar{g}^{00} + \bar{g}_{11}\bar{g}^{10})A(\xi_0) \tag{42}$$

$$g = (\bar{g}_{01}\bar{g}^{01} + \bar{g}_{11}\bar{g}^{11})\frac{ck}{\omega}\frac{dA(\xi)}{d\xi}\bigg|_{\xi=\xi_0} \tag{43}$$

$$d = \frac{v_e}{c}A(\xi_0) \tag{44}$$

$$\xi_0 = kx_0 \tag{45}$$

Since v_e/c is of order 10^{-4}, i.e., $d \sim 10^{-4}$ and $1 + (d \sin \omega t)^2 \to 1$, equation (35) can be reduced to

$$v_y = cd \sin \omega t \left[-\bar{g}_{00} - 2\bar{g}_{01} \frac{v_x}{c} - \bar{g}_{11}\left(\frac{v_x}{c}\right)^2 \right]^{1/2} \tag{46}$$

The velocity of a free election moving in a strong-laser plasma can be written as

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} \tag{47}$$

[(**i**, **j**, **k**) are the unit vectors in the directions of axes (x, y, z)] and then β and $\dot{\beta}$ can be expressed as

$$\beta = \frac{v_x}{c}\mathbf{i} + \frac{v_y}{c}\mathbf{j} \tag{48}$$

$$\dot{\beta} = \frac{\dot{v}_x}{c}\mathbf{i} + \frac{\dot{v}_y}{c}\mathbf{j} \tag{49}$$

$\dot{v}_x$ and $\dot{v}_y$ are the derivatives of the velocity components in the x and y directions with respect to time. From equations (38) and (46), we obtain

$$\dot{v}_x = c\omega(ag - bf)/(f \cos \omega t + g \sin \omega t)^2 \tag{50}$$

$$\begin{aligned} \dot{v}_y = {} & c\, d\omega \cos \omega t \left[-\bar{g}_{00} - 2\bar{g}_{01} \frac{v_x}{c} - \bar{g}_{11}\left(\frac{v_x}{c}\right)^2 \right]^{1/2} \\ & + \frac{1}{2} cd \sin \omega t\left(-2\bar{g}_{01} \frac{\dot{v}_x}{c} - 2\bar{g}_{11} \frac{v_x}{c}\frac{\dot{v}_x}{c} \right) \\ & \times \left[-\bar{g}_{00} - 2\bar{g}_{01} \frac{v_x}{c} - \bar{g}_{11}\left(\frac{v_x}{c}\right)^2 \right]^{-1/2} \end{aligned} \tag{51}$$

Similarly, the unit vector $\mathbf{r}_1$ can be written as

$$\mathbf{r}_1 = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k} \tag{52}$$

where

$$x_1 = (x - x_0)/r \tag{53}$$

$$y_1 = (y - y_0)/r \tag{54}$$

$$z_1 = (z - z_0)/r \tag{55}$$

Inserting equations (48), (49), and (52) into equations (33) and (34) gives the radiation intensity emitted by a free electron in a strong-laser plasma

$$\mathbf{E} = \frac{e}{c}\frac{1}{k^3 r}\left(\left\{y_1\left[\left(x_1 - \frac{v_x}{c}\right)\frac{\dot{v}_y}{c} - \left(y_1 - \frac{v_y}{c}\right)\frac{\dot{v}_x}{c}\right] - z_1^2\frac{\dot{v}_x}{c}\right\}\mathbf{i}\right.$$
$$+ \left\{-x_1\left[\left(x_1 - \frac{v_x}{c}\right)\frac{\dot{v}_y}{c} - \left(y_1 - \frac{v_y}{c}\right)\frac{\dot{v}_x}{c}\right] - z_1^2\frac{\dot{v}_y}{c}\right\}\mathbf{j}$$
$$\left. + \left(x_1 z_1\frac{\dot{v}_x}{c} + y_1 z_1\frac{\dot{v}_y}{c}\right)\mathbf{k}\right) \tag{56}$$

$$\mathbf{B} = \frac{e}{c}\frac{1}{k^3 r}\left\{z_1\left[\frac{\dot{v}_y}{c} - \frac{x_1(v_x\dot{v}_y - v_y\dot{v}_x)}{c^2}\right]\mathbf{i}\right.$$
$$- z_1\left[\frac{\dot{v}_x}{c} + \frac{y_1(v_x\dot{v}_y - v_y\dot{v}_x)}{c^2}\right]\mathbf{j}$$
$$\left. + \left[\frac{y_1\dot{v}_x - x_1\dot{v}_y}{c} + \frac{(y_1^2 + x_1^2)(v_x v_y - \dot{v}_x\dot{v}_y)}{c^2}\right]\mathbf{k}\right\} \tag{57}$$

where

$$k = 1 - \left(x_1\frac{v_x}{c} + y_1\frac{v_y}{c}\right) \tag{58}$$

3.2. Radiation Power

The radiation emitted by an accelerated free electron can be regarded as the coherent superposition of the contributions of two acceleration components respectively parallel and perpendicular to the velocity. Now we decompose the acceleration of a free electron in a strong-laser plasma into two components parallel and perpendicular to the velocity, respectively.

The cosine of the angle ϕ between the velocity and the acceleration is

$$\cos\phi = \frac{v_x\dot{v}_x + v_y\dot{v}_y}{(v_x^2 + v_y^2)^{1/2}(\dot{v}_x^2 + \dot{v}_y^2)^{1/2}} \tag{59}$$

Therefore, the two acceleration components in the directions parallel and perpendicular to the velocity are

$$\dot{v}_{\parallel} = |\dot{\mathbf{v}}|\cos\phi \tag{60}$$

$$\dot{v}_{\perp} = |\dot{\mathbf{v}}|\sin\phi \tag{61}$$

where

$$|\dot{\mathbf{v}}| = (\dot{v}_x^2 + \dot{v}_y^2)^{1/2} \tag{62}$$

$$\sin \phi = \frac{|v_x\dot{v}_y - v_y\dot{v}_x|}{(v_x^2 + v_y^2)^{1/2}(\dot{v}_x^2 + \dot{v}_y^2)^{1/2}} \tag{63}$$

The two components of the acceleration [equations (60) and (61)] can be written as

$$\dot{v}_\parallel = (v_x\dot{v}_x + v_y\dot{v}_y)/(v_x^2 + v_y^2)^{1/2} \tag{64}$$

$$\dot{v}_\perp = |v_x\dot{v}_y - v_y\dot{v}_x|/(v_x^2 + v_y^2)^{1/2} \tag{65}$$

The radiation power emitted by the parallel and the perpendicular acceleration components is

$$P_\parallel = \frac{2}{3}\frac{e^2}{c^3}\dot{v}_\parallel^2\gamma^6 \tag{66}$$

$$P_\perp = \frac{2}{3}\frac{e^2}{c^3}\dot{v}_\perp^2\gamma^4 \tag{67}$$

where

$$\gamma = 1/(1 - B^2)^{1/2} \tag{68}$$

$B = |\beta|$, and β is given by equation (48).

Inserting equations (60) and (61) into equations (62) and (63) yields

$$P_\parallel = \frac{2}{3}\frac{e^2}{c^3}\gamma^6\frac{(v_x\dot{v}_x + v_y\dot{v}_y)^2}{v_x^2 + v_y^2} \tag{69}$$

$$P_\perp = \frac{2}{3}\frac{e^2}{c^3}\gamma^4\frac{(v_x\dot{v}_y - v_y\dot{v}_x)^2}{v_x^2 + v_y^2} \tag{70}$$

From equations (69) and (70), we get

$$\frac{P_\parallel}{P_\perp} = \gamma^2\left(\frac{v_x\dot{v}_x + v_y\dot{v}_y}{v_x\dot{v}_y - v_y\dot{v}_x}\right)^2 \tag{71}$$

In practice, d is of order 10^{-4}, $v_y \sim 10^{-4}v_x$, $\dot{v}_y \sim 10^{-4}v_x$, and $\dot{v}_x \sim v_x$ [from equations (38), (46), (50), and (51)]. Therefore, $v_x\dot{v}_x + v_y\dot{v}_y \sim v_x^2$, $v_x\dot{v}_y - v_y\dot{v}_x \sim 10^{-4}v_x^2$, $(v_x\dot{v}_x + v_y\dot{v}_y)/(v_x\dot{v}_y - v_y\dot{v}_x) \sim 10^4$, and $P_\parallel/P_\perp \sim 10^8\gamma^2$. For a relativistic particle, $\gamma >> 1$ and then $P_\parallel >> P_\perp$. Namely, for a free electron in a strong-laser plasma, the radiation power of the parallel acceleration component is much greater than that of the perpendicular component. So the perpendicular acceleration component can be neglected, and the total radiation power is considered to be emitted by the parallel component. The radiation is of bremsstrahlung type, and its angular distribution is limited to the small cone along the direction of motion.

4. CONCLUSION

We have obtained the four- and three-dimensional velocities of a free electron in a strong-laser plasma by solving the generally covariant equation of motion for an electron. The radiation emitted by a free electron in a strong-laser plasma was studied, and the radiation intensity and the radiation power were derived. The radiation emitted by the acceleration component perpendicular to the velocity of a free electron in a strong-laser plasma can be neglected as compared to that of the parallel component. Then the total radiation power can be approximately considered to be emitted by the parallel component, and the radiation emitted by a free electron in a strong-laser plasma is a sort of bremsstrahlung.

Obviously, if $g_{\alpha\beta} = \eta_{\alpha\beta}$, then the obtained results become the case of flat space-time. Therefore the difference between the cases with and without the medium can be used as a test of the Riemann geometry given in Zhu *et al.* (1995).

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